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R-functions method in problems of temperature fields calculation for fuel rods

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In this paper the numerical solutions of model multiparametrical stationary temperature fields calculation problems for fuel rods of complex cross section, including having a translation type of symmetry, by method of R-functions are considered. It is shown, that the method of R-functions is convenient for the solution of such problems for fuel rods with any cross section, allows to receive necessary accuracy of the solution and gives the user an opportunity of operative change of the geometrical and physical information.

1. Introduction

One of the main problems of thermal reactor calculation is determination of temperature fields in fuel elements. Rather high requirements are showed to fuel elements concerning their reliability, which substantially depends on a correct choice of its temperature mode. It is obvious, that calculation of temperature fields in fuel elements is very important for a correct choice of their designs and allowable capacities of heat generation.

The transfer of heat in fuels rods is mainly carried out by thermal conductivity, therefore calculation of temperature fields for fuel rods is reduced to the solution of thermal conductivity problems at presence of internal sources of heat. However only few thermal conductivity problems in elements of a reactor’s structure allow the analytical solution. Complex form of elements, non-homogeneous boundary conditions, dependence of thermal conductivity capacity on coordinates and time (for non-stationary problems), necessity for many cases to take into account dependence of physical properties of a material on temperature - all this complicates or makes impossible to use analytical calculation methods. For their solution the numerical methods can be used.

Recently for the solution of boundary thermal conductivity problems in increasing frequency the approximate analytical methods are applied, including direct methods, for example, variational. However during long time the application of variational methods was complicated because of impossibility of construction of coordinate functions precisely satisfying to boundary conditions for areas of the complex form and having property of completeness.

V. L. Rvachev with the help of the theory of R-functions has solved a problem of construction of complete systems of coordinate functions for areas of the complex form and various types of boundary conditions, that in turn has enabled essentially to expand application of variational methods in practice. The constructive tool of a method of R-functions allows to take into account the geometrical information at an analytical level without any approximation and solves a problem of satisfaction to boundary conditions of the most various types for areas practically of any form.

The problem important today is the creation of computer models of physical processes allowing, varying geometrical and physical parameters, to do multiversion calculations.

2. Purpose

The purpose of this paper is the numerical solution of model multiparametrical stationary temperature fields calculation problems for fuel rods by a method of R-functions.

3. Solution of model problems

On stationary operating conditions temperature fields in them at constant thermal conductivity λ and packed thermal conductivity density q_V are described by the Poisson equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=-\frac{q_V(x,y,z)}{\lambda},$$

which at absence of heat sources becomes the Laplace equation (here u– temperature).

In nuclear reactors of various purpose fuel rods with cross sizes much less than their lengths are applied. At calculation of temperature fields in such fuel elements in many problems it is possible to neglect a thermal flow lengthways the fuel rod and to consider a field of temperatures two-dimensional, satisfying in area Ω, which is a cross section of fuel element, Poisson equation

$$\Delta u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-f(x,y), \tag{1}$$

where $f(x,y)=\dfrac{q_v(x,y)}{\lambda}$. Heat is taken off a fuel element's surface by coolant having weight-average temperature u_L. The boundary conditions are conditions of the third type of convection heat exchange between a surface and environment

$$\lambda\frac{\partial u}{\partial n}+\alpha_s(u-u_L)=0, \tag{2}$$

where α_s– heat-transfer coefficient. It may change along fuel element border and be equal to zero on some sites (for example, in points of a two fuel elements contact).

Let's rewrite the boundary condition (2) as following

$$\frac{\partial u}{\partial n}+h(x,y)u\big|_{\partial\Omega}=\psi(x,y), \tag{3}$$

where $h(x,y)=\dfrac{\alpha_s}{\lambda}, \psi(x,y)=\dfrac{\alpha_s}{\lambda}u_L$.

Then according to a method of R-functions, the solution of a problem (1) - (3) is represented as structure

$$u=P_1-\omega D_1 P_1+hP_1\omega+\psi\omega+\omega^2 P_2, \tag{4}$$

where $\omega=\omega(x,y)=0$ – the normalized equation of border of area $\partial\Omega$, $P=(P_1,P_2)$ – undefined component of solution structure – is represented as

$P_1 = \sum_{i=1}^{N} C_i \chi_i$, $P_2 = \sum_{i=1}^{N} C_{i+N} \chi_i$. Here $\chi_i = \chi_i(x, y)$ – complete system of coordinate functions. The operator D_1 is used for continuation of boundary conditions inside of area Ω. Further, for finding undefined components we apply a Ritz method. For this purpose we transit to a boundary problem with homogeneous boundary conditions and on lineal of functions, satisfying them, we shall construct functional, equivalent to the given boundary problem. The transition to homogeneous boundary conditions is carried out by replacement $u = u_0 + u_1$, where u_0 satisfies boundary conditions (3). Then the equivalent variational problem is to minimize of the following functional:

$$J(u) = \iint_{\Omega} [(\nabla u_1)^2 + 2(\nabla u_1, \nabla u_0) - 2fu_1] d\Omega + \int_{\partial\Omega} (hu_1^2 - 2(\psi - hu_0)u_1) d\Omega.$$

As we are interested in minimum of this function, the coefficients $C_1, C_2, ..., C_{2N}$ must satisfy to system of the equations:

$$\frac{\partial J}{\partial C_j} = \iint_{\Omega} [\nabla u_{1i} \nabla u_{1j} - fu_{1i} + \nabla u_0 \nabla u_{1i}] d\Omega + \int_{\partial\Omega} (hu_{1i} u_{1j} - (\psi - hu_0) u_{1j}) d\Omega, j = \overline{1,2N}.$$

From this linear system, which is called Ritz system, coefficients $C_1, C_2, ..., C_{2N}$ are determined, and, accordingly, the approximate solution u.

Example 1.

A fuel element with cross section as correct hexagon is considered.

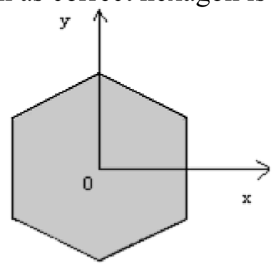


Fig.1.Cross section of a fuel element.

Heat generation is considered constant on area, and both heat emission coefficient and temperature of a liquid – are constant along border, i.e. in a problem (1) - (3):

$$f = \frac{q_s}{\lambda} = const, h = \frac{\alpha_s}{\lambda} = const, \psi = \frac{\alpha_s}{\lambda} T_L = hT_L = const .$$

The normalized equation of area border $\partial\Omega$ ($\omega = 0$) was constructed in the form:

$$\omega = \frac{\omega_1}{1 + \omega_1^{\ 2}},$$

$$\omega_1(x, y) = \sigma_0(\rho \cos \mu_n(\theta, m), y), \rho = \sqrt{x^2 + y^2}, \theta = arctg\frac{y}{x},$$

$$\mu_n(\theta, m) = \frac{8}{m\pi} \sum_{i=1}^{n} \frac{(-1)^{i+1}}{(2i-1)^2} \sin\left[\frac{(2i-1)m\theta}{2}\right], \ \sigma_0 \equiv (r - x) \geq 0 .$$

In a fig. 2 the distribution of temperature in section $y = 0$ is given at various values of h, which actually defines the character of conformity between temperature conditions in an environment and distribution of temperature in a body.

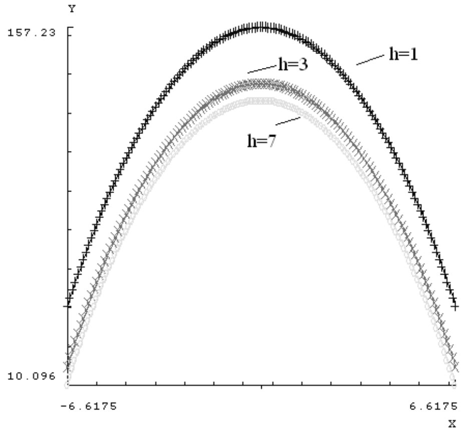


Fig.2. Distribution of temperature depending on parameter h.

As we see, at increase of parameter h, temperature in fuel element is reduced.

Example 2.

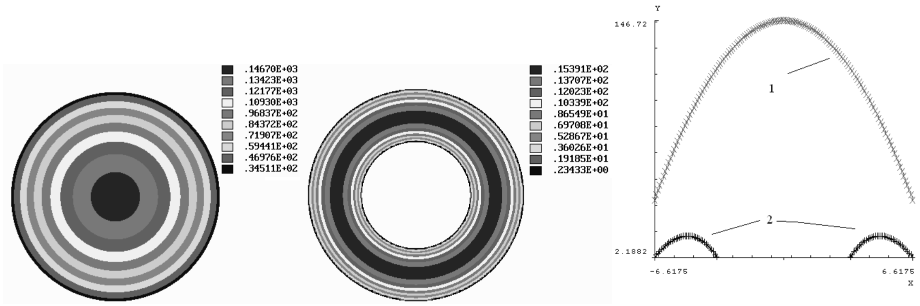
At constructing of fuel elements they not only choose materials with good thermophysical and nuclear properties, but also aspire to ensure good heat transfer between fuel element and coolant. The improvement of heat transfer, for example, is reached at the expense of increasing the relation of a surface to volume of fuel element. Let's consider the solution of a problem (1) - (3) with the same conditions, as well as in the previous problem, in cylindrical (section - circle) and tubular (section - ring) fuel elements, which are the most widespread forms. The equations of borders of areas in this case were constructed as follows:

$\omega = (r^2 - x^2 - y^2)/2r = 0$ - the normalized equation of border of area of cylindrical fuel element section;

$\omega = ((r^2 - x^2 - y^2)/2r) \wedge \overline{(((r/2)^2 - x^2 - y^2)/2(r/2)))} = 0$ - the normalized equation of border of area of tubular fuel element section.

In a fig. 3 the comparison of distribution of temperature in fuel elements is represented

In fuel element, which section represents a ring, at the expense of the large relation of a surface to volume, heat transfer is better and in whole temperature inside fuel lower.



a) *b)* *c)*

Fig.3. a) picture of level lines of a temperatures field in cylindrical fuel element; b) picture of level lines of a temperatures field in tubular fuel element; c) the plots of the solutions in section $y = 0$ for cylindrical (1) and tubular (2) fuel elements.

Example 3.

The cross section of fuel element represents correct hexagon with 91 symmetrically located circular apertures.

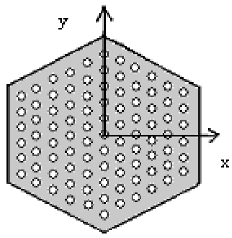


Fig.4. Cross section of fuel element with cylindrical channels.

The intensity of heat generation is considered constant on section. All formed heat is removed by a liquid proceeding in apertures, i.e. on circles the heat exchange is given. On the sides of hexagon thermal flows are equal 0 ($\frac{\partial u}{\partial n} = 0$).Temperature of a liquid in all apertures assume identical, therefore in a boundary condition (3) we accept $u_L = 0$, i.e. we count temperature in fuel element from temperature of a liquid.

Then in a boundary condition (3) $\psi = 0$, and $h = \dfrac{\omega_1 \frac{\alpha}{\lambda}}{\omega_1 + \omega_2}$ - is received under conglutination formula

The normalized equation of area border $\partial\Omega$ ($\omega = 0$), the normalized equation of a site of border $\partial\Omega_1$ ($\omega_1 = 0$), on which the thermal flow is given, the normalized equation of a site of border $\partial\Omega_2$ ($\omega_2 = 0$), on which the heat exchange is given, were constructed using following formulas:

$$\omega = \omega_1 \wedge \omega_2,$$

$$\omega_1(x, y) = \sigma_0(\rho \cos \mu_n(\theta, m), y), \rho = \sqrt{x^2 + y^2}, \theta = arctg\frac{y}{x},$$

$$\sigma_0 \equiv [(r-x) \ge 0],$$

$$\mu_n(\theta, m) = \frac{8}{m\pi} \sum_{i=1}^{n} \frac{(-1)^{i+1}}{(2i-1)^2} \sin\left[\frac{(2i-1)m\theta}{2}\right],$$

$$\omega_2(x,y) = \overline{\sigma_1(\mu_n(x,hx), \mu_n(y,hy)) \vee \sigma_2(\mu_n(x,hx), \mu_n(y,hy))},$$

$$\mu_n(x,h) = \frac{4h}{\pi^2} \sum_{i=1}^{n} \frac{(-1)^{i+1}}{(2i-1)^2} \sin\left[\frac{(2i-1)x\pi}{h}\right],$$

$$\sigma_1 \equiv [(R^2 - x*x - y*y)/(2R\ ) \ge 0,$$

$$\sigma_2 \equiv [(R^2 - (x - hx/2)*(x - hx/2) - (y - hy/2)*(y - hy/2))/(2R) \ge 0].$$

It is necessary to note, that the construction of an area border function with the help of translation has allowed to apply R - operations only 2 times, instead of 94 times by quantity of channels (91), i.e. to automate process of construction compound area border functions.

It would be desirable to note, that the problem was solved in two ways - two types of structures were used: 1) structure with natural boundary conditions $u = P_1$; 2) structure (4), structure precisely satisfying all boundary conditions. In the first case the necessary accuracy of the solution was achieved by increase of dimension of a splines grid, in the second case - at the expense of exact satisfaction to boundary conditions. In the first case linear splines were used, and in second cubic splines B_3. The results received by both ways, coincide, that also can be one of confirmations of their reliability.

In a fig.5 the picture of level lines of a temperature field is represented.

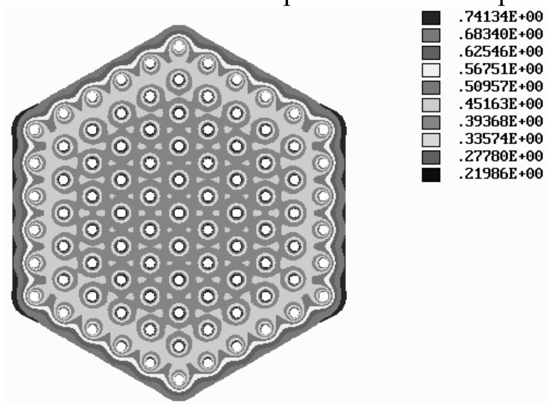


Fig.5. Distribution of temperature in fuel element.

The results of calculations have shown, that at considered homogeneous on fuel element section distribution of apertures there is an essential change of temperature on directions from the centre of fuel element to its sides.

It is possible to allocate two categories of channels in fuel element which are not in equivalent conditions from the point of view of cooling by the coolant: channels of the central and peripheral zone. The channels of the central zone are surrounded with identical cells, and the peripheral channels located at flat sides, are non-uniformly cooled on perimeter because of various configuration of cells around them. In result temperature essentially varies on perimeter of peripheral elements. Also heat is removed more through peripheral channels than through central.

The computer modeling was spent under operating conditions of POLYE system, which was developed in A.N. Podgorny Institute for problems in machine building under the direction of the academician of Ukrainian NAS V. L. Rvachev. This system allows to solve boundary problems with various types of boundary conditions for areas of any form, in short term to provide a plenty of numerical experiments and operatively to solve problems of calculation of fields of a various physical nature.

5. Resume

The numerical solutions of model multiparametrical problems of calculation of stationary temperature fields for fuel rods of complex section are considered, that has allowed to choose the appropriate techniques for the subsequent solution of real problems.

Is shown, that the method of R-functions is convenient for the solution of problems of calculation of fields of temperature in fuel rods with any cross section, allows to receive necessary accuracy of the solution and gives the user an opportunity of operative change of the geometrical and physical information. It enables to experiment with the mathematical model, to vary parameters and to play with the help of model the most various situations.

LITERATURE

1. Федик И.И., Колесов В.С., Михайлов В.Н. Поля и термонапряжения в ядерных реакторах. — М.:Энергоатомиздат, 1985. — 280 с.
2. Петухов Б.С. Теплообмен в ядерных энергетических установках. — М.:Энергоатомиздат, 1986. — 472 с.
3. Михлин С. Г. Вариационные методы математической физики. — М., Наука, 1970. – 512 с.
4. Рвачев В.Л. Теория R-функций и некоторые её приложения. Киев, Наук. думка, 1982.— 552 с.
5. Рвачев В.Л. Проблемно-ориентированные языки и системы для инженерных расчетов / В.Л.Рвачев, А.Н.Шевченко.— Киев: Техніка, 1988. — 197 с.
6. Рвачев В.Л. Метод R-функций (RFM) в краевых задачах с геометрической и физической симметрией / Рвачев В.Л., Шапиро В., Шейко Т.И. // Математичні методи та фізико-механічні поля. — 41, №1, 1998. — С. 146-159.
7. Рвачев В.Л. Применение метода R-функций к построению уравнений локусов, обладающих симметрией / Рвачев В.Л., Шапиро В., Шейко Т.И. // Электромагнитные волны и электронные системы. — т.4, № 4, 1999. — С. 4-20.
8. Максименко-Шейко К.В. R-функции в математическом моделировании геометрических объектов, обладающих симметрией /К.В.Максименко-Шейко, Т.И.Шейко // Кибернетика и системный анализ. — №6, 2008. — С.75-83.