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Discrete mathematical model of diffraction on periodic pre-Cantor gratings with shield and numerical experiment

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Discrete mathematical model of the diffraction problem of a plane electromagnetic wave on a periodic pre-fractal grating consisting of perfectly conducting strips located above the shield based on the boundary of singular integral equations in case of H polarized had been created. The numerical solution method of these problems based on the created discrete mathematical model is proposed. Plots of dependence fields of the grating order, distance to shield, incidence angle have been constructed.

Key words: *diffraction problem, numerical experiment, boundary value problem, boundary singular integral equations.*

Построена дискретная математическая модель задач дифракции плоской электромагнитной волны на периодической предфрактальной решетке, состоящей из идеально проводящих лент, расположенных над отражателем, на базе граничных сингулярных интегральных уравнений в случае H поляризации. Предлагается метод численного решения указанной задачи на базе построенной дискретной математической модели. Построены графики зависимостей полей от порядка решетки, расстояния до экрана, угла падения.

Ключевые слова: *задача дифракции, численный эксперимент, краевая задача, граничное сингулярное интегральное уравнение.*

Побудовано дискретну математичну модель задачі дифракції плоскої електромагнітної хвилі на періодичній предфрактальній решітці, що складається з ідеально провідних стрічок, розташованих над відбивачем, на базі граничних сингулярних інтегральних рівнянь у випадку H - поляризації. Пропонується метод чисельного розв’язку вказаних задач на базі побудованих дискретних математичних моделей. Побудовані графіки залежностей полів від порядку решітки, відстані до екрану, кута падіння.

Ключові слова: *задача дифракції, чисельний експеримент, крайова задача, граничне сингулярне інтегральне рівняння.*

1. Introduction

In this paper we consider the problem of diffraction of a plane monochromatic electromagnetic wave on a periodic "pre-fractal grating" consisting of perfectly conducting strips located above a perfectly conducting shield.

Pre-Cantor set of intervals is set of intervals $L_{2l}^{(N)}$, obtained by the principle of constructing a Cantor set in each period $[-l,l]$ on N step (see Fig.1) [6].

Denote periodic grating, which is in the plane of section YZ, in case of the N-th step of the construction of Cantor set [6]:

$$L_{2l}(N)=\left\{(y,z)\in\Re^2, y\in\bigcup_{r=-\infty}^{+\infty}\bigcup_{q=1}^{2^N}\left(a_q^N+2l\cdot r, b_q^N+2l\cdot r\right),\ z=0\right\},$$
$$-l<a_1^N<b_1^N<\ldots<a_{2^N}^N<b_{2^N}^N<l, \tag{1.1}$$

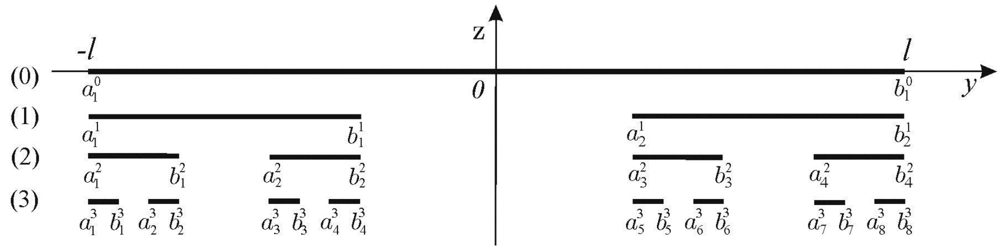

Fig.1.1. Pre-Cantor sets $L_{2l}^{(0)}, L_{2l}^{(1)}, L_{2l}^{(2)}, L_{2l}^{(3)}$.

Fractal grating are used as antennas and are wideband and multi-band.

The numerical solution of discrete singularities method of the wave diffraction problem on pre-fractal gratings without shield had been carried out in work [7].

The full electromagnetic field is represented as a superposition of two fields: $(E_x,0,0)$, $(0,H_y,H_z)$ is E polarization and $(0,E_y,E_z)$, $(H_x,0,0)$ is H polarization, for solving the two-dimensional (2D) problems of the mathematical theory of diffraction. In this case the stationary Maxwell equations (time dependence is given by the factor $e^{-i\omega t}$) are reduced to two stationary wave equations. The complex amplitude of the desired fields that depend on spatial coordinates $E_x = u(y,z)$ or $H_x = u(y,z)$ respectively satisfy the Helmholtz equation above the shield without the strips:

$$\Delta u(y,z) + k^2 u(y,z) = 0, \quad k = \frac{\omega}{c}, \tag{1.2}$$

boundary conditions on the strip and the shield, Summerfeld radiation conditions and Meixner condition at the edges of the strips. In the case of E polarization of the problem reduces to Dirichlet boundary value problem, as in the case of H polarization of Neumann boundary problem [1-2], which is discussed in this article.

As shown in [2-3], boundary-value problem leads to a boundary singular integral equation with additional conditions on system of intervals.

Effective discrete singularities method (DSM) [2, 4-5] for the numerical solution of a singular equation with additional conditions had been used.

The purpose of this article is to construct a mathematical model of the diffraction problem of a monochromatic plane H polarized [11] wave on a periodic pre-Cantor grating which lies over shield on the basis of a singular integral equation for a system of intervals with additional conditions, and on this basis the discrete mathematical model of the diffraction problem have been constructed using the DSM [7].

2. Formulation of the boundary-value problem for Helmholtz equation

We choose a Cartesian coordinate system so that grating lay in the the XY plane and the X-axis is parallel to the edges of strips. In plane $z=-h$ of an infinite perfectly conducting screen, above which is located a periodic system $L_{2l}(5)$ of infinitely thin perfectly conducting strips, where $z>0$ and $-h<z<0$ when $\varepsilon_0 = 1$ is dielectric constant.

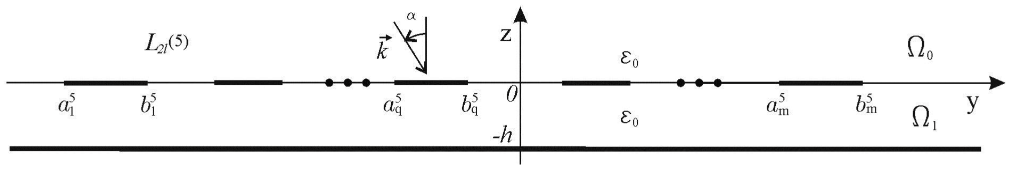

Fig.2.1. Cross section of the considered diffraction structure in the YZ plane.

Let H polarized plane electromagnetic wave obliquely incident from infinity to top of the diffraction structure

$$u_{inc}^{(N)}(y,z) = H_x(y,z) = e^{ik(y\sin\alpha - z\cos\alpha)}. \tag{2.1}$$

Need to find the full field resulting from the scattering of waves on the consideration of diffraction structure (Fig. 2.1).

The full field found in the form

$$u^{(N)}(y,z) = \begin{cases} u_0^{(N)}(y,z) + u_+^{(N)}(y,z), & z > 0, \\ u_0^{(N)}(y,z) + u_-^{(N)}(y,z), & -h < z < 0, \end{cases} \tag{2.2}$$

where $u_0^{(N)}(y,z)$ is known solution of Helmholtz equation, represents the sum of incident and reflected waves from the shield (no strips)

$$u_0^{(N)}(y,z) = u_{0,inc}^{(N)}(y,z) + Au_{0,ref}^{(N)}(y,z),$$

where A is reflection coefficient and $u_+^{(N)}(y,z)$, $u_-^{(N)}(y,z)$ will be determined.

Full field $u^{(N)}(y,z)$ should satisfy the following conditions:

- Helmholtz equation (1.2) above the shield without the strips.
- Boundary conditions on the strips

$$\frac{\partial u^{(N)}(y,z)}{\partial z}\Big|_{z=0} = 0, \quad y \in L_{2l}^{(N)}, \tag{2.3}$$

and on the shield

$$\frac{\partial u^{(N)}(y,z)}{\partial z}\Big|_{z=-h} = 0, \quad y \in \Re. \tag{2.4}$$

- The conditions of conjugation in the gaps

$$u^{(N)}(y,+0) = u^{(N)}(y,-0), \quad y \in CL_{2l}^{(N)} = [-l,l] \setminus L_{2l}^{(N)}, \tag{2.5}$$

$$\frac{\partial u^{(N)}}{\partial z}(y,+0) = \frac{\partial u^{(N)}}{\partial z}(y,-0), \quad y \in CL_{2l}^{(N)}. \tag{2.6}$$

- Sommerfeld radiation condition at infinity.
- The condition of Meixner on the edges of strips.

The field $u_0^{(N)}(y,z)$ has the form

$$u_0^{(N)}(y,z) = 2\cos(k(z+h)\cdot\cos\alpha)\cdot e^{iky\sin\alpha}, \quad z > -h,\ y \in \Re. \tag{2.7}$$

The fields $u_{+}^{(N)}(y,z),\, z>0$ and $u_{-}^{(N)}(y,z),\, -h<z<0$ found in the form Fourier-representations which satisfy the Helmholtz equation in the correspond fields:

$$u_{+}^{(N)}(y,z)=\sum_{s=-\infty}^{+\infty} C_s^{(N)} e^{ik_s y-\gamma(k_s)z}, \quad z>0, \tag{2.8}$$

$$u_{-}^{(N)}(y,z)=-\sum_{s=-\infty}^{+\infty} C_s^{(N)} \frac{ch(\gamma(k_s)(z+h))}{sh(\gamma(k_s)h)} e^{ik_s y}, \quad -h<z<0, \tag{2.9}$$

where

$$\gamma_s=\gamma(k_s)=\sqrt{k_s^2-k^2},\, k_s=\lambda_s+k\sin\alpha,\, \lambda_s=\frac{\pi s}{l},\, s\in Z,\, \mathrm{Re}(\gamma_s)\ge 0,\, \mathrm{Im}(\gamma_s)\le 0,$$

this choice radical branch ensures that the of Sommerfeld radiation conditions had been implemented.

3. The boundary singular integral equation (SIE)

Introduce the dimensionless variables going back to the period 2π :

$$\xi=\frac{\pi}{l}y,\, \zeta=\frac{\pi}{l}z,\, \kappa=\frac{l}{\pi}k,\, \widetilde{h}=\frac{\pi}{l}h,\, \widetilde{k}_s=\frac{k_s\cdot l}{\pi},\, \widetilde{\gamma}_s=\frac{\gamma_s\cdot l}{\pi},\, a_q^N=\frac{\pi}{l}a_q^N,\, b_q^N=\frac{\pi}{l}b_q^N.$$

Get a pair summation equation for the period $[-\pi,\pi]$ from the boundary conditions (2.3) - (2.4) and conjugation conditions (2.5) - (2.6), taking into account the change of variables entered:

$$\sum_{s=-\infty}^{+\infty} C_s^{(N)}\cdot\left(1+cth\left(\widetilde{\gamma}_s\cdot\widetilde{h}\right)\right)\cdot e^{i\cdot\widetilde{k}_s\cdot\xi}=0, \quad \xi\in CL_{2\pi}^{(N)}, \tag{3.1}$$

$$\sum_{s=-\infty}^{+\infty} \widetilde{\gamma}_s\cdot C_s^{(N)}\cdot e^{i\cdot\widetilde{k}_s\cdot\xi}=f^{(N)}(\xi), \quad \xi\in L_{2\pi}^{(N)}. \tag{3.2}$$

Using the parametric representation of operator Hilbert for periodic function $G(\xi)=G(\xi+2\pi),\, \xi\in\Re$:

$$G(\xi)=\sum_{s=-n}^{n} A_s e^{is\xi},$$

$$\frac{1}{2\pi}\int_{L_{2\pi}^{(N)}} ctg\frac{\xi-\eta}{2}G(\xi)d\xi=\sum_{\substack{s=-n,\\ s\ne 0}}^{n} i\frac{|s|}{s}A_s e^{is\eta}. \tag{3.3}$$

reduce pair summation (3.1) - (3.2) to the SIE as is done in [4 – 5, 8-9].

Denote,

$$U^{(N)}(\xi)=u^{(N)}(\xi,\zeta)\Big|_{\zeta=0}=\sum_{s=-\infty}^{+\infty} C_s^{(N)}\cdot\left(1+cth\left(\widetilde{\gamma}_s\cdot\widetilde{h}\right)\right)\cdot e^{i\cdot(s+\kappa\cdot\sin\alpha)\cdot\xi},$$

$$U^{(N)}(\xi)=0, \quad \xi\in CL_{2\pi}^{(N)},$$

$$\left(U^{(N)}(\xi)\right)' = F^{(N)}(\xi) = \sum_{\substack{s=-\infty \\ s\neq 0}}^{+\infty} i\cdot\tilde{k}_s \cdot C_s^{(N)}\left(1+cth\left(\tilde{\gamma}_s\cdot\tilde{h}\right)\right)\cdot e^{i\cdot\tilde{k}_s\cdot\xi} = 0, \quad \xi\in CL_{2\pi}^{(N)}. \quad (3.4)$$

Considering (3.1) the function $F^{(N)}(\xi)$ satisfy the conditions $F^{(N)}(\xi)=0, \xi\in CL_{2\pi}^{(N)}$;

$$\int_{\alpha_q^N}^{\beta_q^N} F^{(N)}(\eta)d\eta = 0, \quad q=1,...,2^N. \quad (3.5)$$

From definition function $F^{(N)}(\xi)$ and formula (3.5) obtained the integral representation for the coefficients $C_s^{(N)}$ via the function $F^{(N)}(\xi)$:

$$C_s^{(N)} = \frac{1}{2\pi i\cdot\tilde{k}_s\cdot\left(1+cth\left(\tilde{\gamma}_s\cdot\tilde{h}\right)\right)}\cdot\int_{L_{2\pi}^{(N)}} F^{(N)}(\eta)\cdot e^{-i\cdot\tilde{k}_s\cdot\eta}d\eta, \quad (3.6)$$

$$\tilde{\gamma}_s^{(N)} = \tilde{W}_s^{(N)} + O\left(\frac{1}{s}\right), \tilde{W}_s^{(N)} = |s| + \frac{|s|}{s}\kappa\sin\alpha, \Delta_s^{(N)} = \tilde{\gamma}_s^{(N)} - \tilde{W}_s^{(N)} = O\left(\frac{1}{s}\right), s\to\infty. \quad (3.7)$$

By virtue of (3.3), (3.6) ,(3.7) from (3.2) we obtained:

$$-\frac{1}{2\pi}\int_{L_{2\pi}^{(N)}} e^{i\cdot\kappa\cdot\sin\alpha\cdot(\xi-\eta)} ctg\frac{\eta-\xi}{2}F^{(N)}(\eta)d\eta +$$

$$+\frac{1}{\pi}\int_{L_{2\pi}^{(N)}}\sum_{s=-\infty}^{+\infty}\frac{\Delta_s^{(N)}}{2i\cdot\tilde{k}_s}e^{i\cdot\tilde{k}_s\cdot(\xi-\eta)}F^{(N)}(\eta)d\eta = f^{(N)}(\xi), \xi\in L_{2\pi}^{(N)}. \quad (3.8)$$

Move on to SIE with the Cauchy kernel, denote in (3.8)

$$\tilde{K}^{(N)}(\xi,\eta) = \left(\frac{1}{2}e^{i\cdot\kappa\cdot\sin\alpha\cdot(\xi-\eta)}\cdot ctg\frac{\eta-\xi}{2} - \frac{1}{\eta-\xi}\right) - \sum_{s=-\infty}^{+\infty}\frac{\Delta_s^{(N)}}{2i\cdot\tilde{k}_s}\cdot e^{i\cdot\tilde{k}_s\cdot(\xi-\eta)}. \quad (3.9)$$

Write the obtained SIE with the Cauchy kernel considering (3.8) with additional conditions (3.5)

$$\begin{cases} \frac{1}{\pi}\int_{L_{2\pi}^{(N)}}\frac{F^{(N)}(\eta)}{\eta-\xi}d\eta + \frac{1}{\pi}\cdot\int_{L_{2\pi}^{(N)}}\tilde{K}^{(N)}(\eta,\xi)\cdot F^{(N)}(\eta)d\eta = f^{(N)}(\xi), \quad \xi\in L_{2\pi}^{(N)}, \\ \frac{1}{\pi}\int_{L_p^{(N)}} F^{(N)}(\eta)d\eta = 0, \quad L_p^{(N)} = \left(\alpha_p^N, \beta_p^N\right), \quad p=1,...,2^N. \end{cases} \quad (3.10)$$

4. The transition to the systems of singular integral equations (SSIE) with additional conditions on a standard interval (-1,1)

Introduce restriction of the function $F^{(N)}(\eta)$ on intervals $L_q^{(N)} = \left(\alpha_q^N, \beta_q^N\right), q=1,...,2^N$:

$$F_q^{(N)}(\eta) = F^{(N)}(\eta)|_{\eta \in L_q^{(N)}},$$
$$f_p^{(N)}(\xi) \equiv f^{(N)}(\xi)|_{\xi \in L_p^{(N)}}, \xi \in L_p^{(N)}.$$

Meixner condition had been done if function $F_q^{(N)}(\eta)$ represented as

$$F_q^{(N)}(\eta) = \frac{w_q^{(N)}(\eta)}{\sqrt{\left(\beta_q^N - \eta\right)\cdot\left(\eta - \alpha_q^N\right)}}, \eta \in L_q^{(N)}. \tag{4.1}$$

Choose a standard interval (-1,1) and display it on the intervals $L_q^{(N)}, q = \overline{1,2^N}$:

$$g_q^{(N)} : (-1,1) \mapsto \left(\alpha_q^N, \beta_q^N\right):$$
$$t \mapsto g_q^{(N)}(t) = \frac{\beta_q^N - \alpha_q^N}{2} t + \frac{\beta_q^N + \alpha_q^N}{2}, \quad -1 < t < 1,$$

then function (4.1) has the from

$$F_q^{(N)}(\eta) = F_q^{(N)}\left(g_q^{(N)}(t)\right) = \frac{2 \cdot v_q^{(N)}(t)}{(\beta_q^N - \alpha_q^N) \cdot \sqrt{1-t^2}}, \quad -1 < t < 1. \tag{4.2}$$

Denote

$$v_q^{(N)}(t) = u_q^{(N)}\left(g_q^{(N)}(t)\right),$$
$$\tilde{f}_p^{(N)}(t_0) = f_p^{(N)}\left(g_p^{(N)}(t_0)\right),$$
$$K_{qp}^{(N)}(t, t_0) = \tilde{K}_{qp}^{(N)}\left(g_q^{(N)}(t), g_p^{(N)}(t_0)\right).$$

Performing the transformation carried out in [4 – 5, 10], from (3.10) obtain

$$\begin{cases} \dfrac{1}{\pi}\displaystyle\int_{-1}^{1} \frac{v_p^{(N)}(t)}{t - t_0} \cdot \frac{dt}{\sqrt{1-t^2}} + \frac{1}{\pi}\sum_{q=1}^{2^N}\int_{-1}^{1} K_{qp}^{(N)}(t, t_0) \frac{v_q^{(N)}(t)}{\sqrt{1-t^2}} dt = \tilde{f}_p^{(N)}(t_0), \quad -1 < t_0 < 1, \\ \dfrac{1}{\pi}\displaystyle\int_{-1}^{1} \frac{v_p^{(N)}(t)}{\sqrt{1-t^2}} dt = 0, \quad p = 1,\dots,2^N, \end{cases} \tag{4.3}$$

where $K_{qp}^{(N)}(t, t_0), \tilde{f}_p^{(N)}(t_0)$ is known functions.

5. Discrete mathematical model

The discretization SSIE with the additional conditions (4.3) had been carried out by discrete singularities method [2, 4-5, 7-10]. Transition to SSIE which is used to construct the discrete mathematical model substituting continuous kernel by the interpolation polynomial *(n-2)* and *(n-1)* degree on correspond variables and known function by interpolation polynomial *(n-1)* degree.

Continuous and singular part of equations in (4.3) approximate interpolation quadrature formulas by the two system nodes which represent the zeros of the Chebyshev I and II. Obtained an approximate SSIE interpolating unknown functions $v_p^{(N)}(t)$ polynomials Lagrange's $v_{p,(n-1)}^{(N)}(t),\ p=1,...,2^N$ the degree of (n-1) on n nodes, coinciding with the zeros of Chebyshev polynomials of type I. The problem is reduced to the system of linear algebraic equations (SLAE) to determine the approximate values $v_{p,(n-1)}^{(N)}(t_k^n),\ p=1,...,2^N$ of the function $v_p^{(N)}(t)$ in (4.3) for the function $F_q^{(N)}(\xi)=F^{(N)}(\xi)\Big|_{L_q^{(N)}},\ q=\overline{1,2^N}$ using the quadrature formula.

Obtained the SLAE

$$\begin{cases} \dfrac{1}{n}\sum\limits_{k=1}^{n}\dfrac{v_{p,(n-1)}^{(N)}\left(t_k^n\right)}{t_k^n-t_{0j}^n}+\dfrac{1}{n}\sum\limits_{q=1}^{2^N}\sum\limits_{k=1}^{n}K_{qp}^{(N)}\left(t_k^n,t_{0j}^n\right)v_{q,(n-1)}^{(N)}\left(t_k^n\right)=\tilde{f}_p^{(N)}\left(t_{0j}^n\right), \\ \qquad\qquad j=\overline{1,n-1},\ p=\overline{1,2^N}, \\ \sum\limits_{k=1}^{n}v_{p,(n-1)}^{(N)}\left(t_k^n\right)=0, \quad p=1,...,2^N. \end{cases} \tag{5.1}$$

where $t_k^n=\cos\dfrac{2k-1}{2n}\pi,\ k=1,...,n$ is zeros polynomial Chebyshev I of n degree, $t_{0j}^n=\cos\dfrac{j}{n}\pi,\ j=1,...,n-1$ is zeros polynomial Chebyshev II of (n-1) degree.

Form for coefficients $C_s^{(N)}$ have been finally obtained:

$$C_s^{(N)}=\frac{1}{2i\cdot\tilde{k}_s\cdot\left(1+cth\left(\tilde{\gamma}_s\tilde{h}\right)\right)\cdot n}\cdot\sum_{q=1}^{2^N}\sum_{k=1}^{n}v_{q,(n-1)}^{(N)}\left(g_q\left(t_k^n\right)\right)\cdot e^{-i\cdot\tilde{k}_s\cdot g_q\left(t_k^n\right)}. \tag{5.2}$$

6. The results of numerical experiment

In Fig. 6.1 shows plots of dependence the absolute values of complex amplitudes $C_s^{(N)}$ as a function of third order pre-Cantor grating, obtained by numerical solution of linear algebraic equation (5.1) and calculating the coefficients of (5.2).

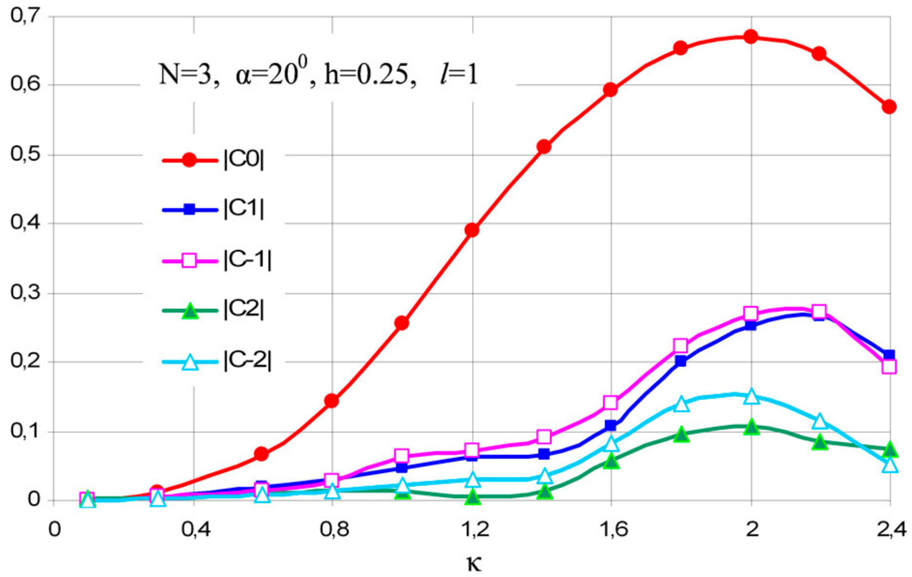

Fig.6.1. Dependence the absolute values of $C_s^{(N)}$ as a function of $\kappa = 2l/\lambda$.

In Fig. 6.2, 6.3 and 6.4 show plots of dependence the absolute values of complex amplitudes $C_s^{(N)}$, s=0,1,2 as a function of the order pre-Cantor grating

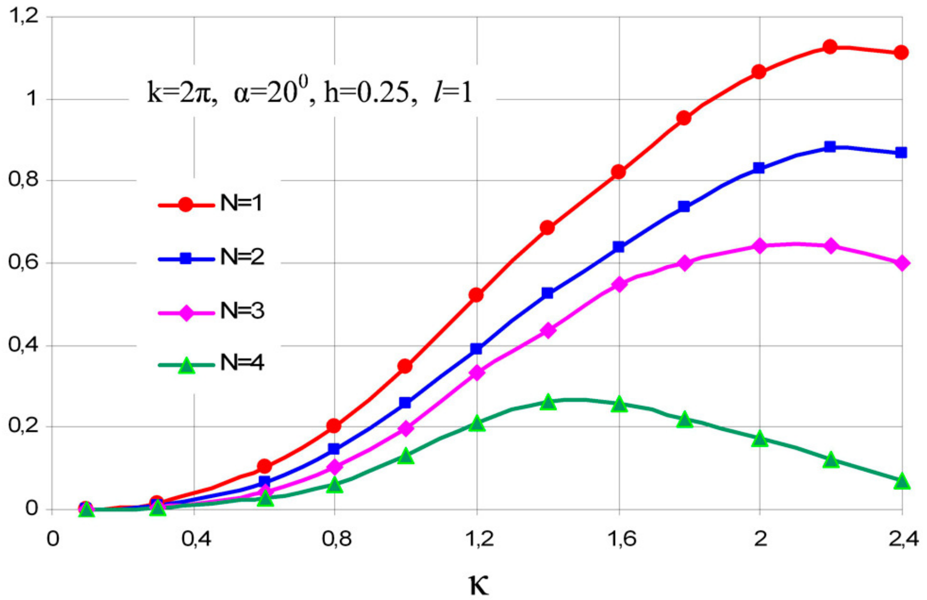

Fig.6.2. Dependence the absolute values of $C_0^{(N)}$ as a function of $\kappa = 2l/\lambda$ for N order pre-Cantor grating.

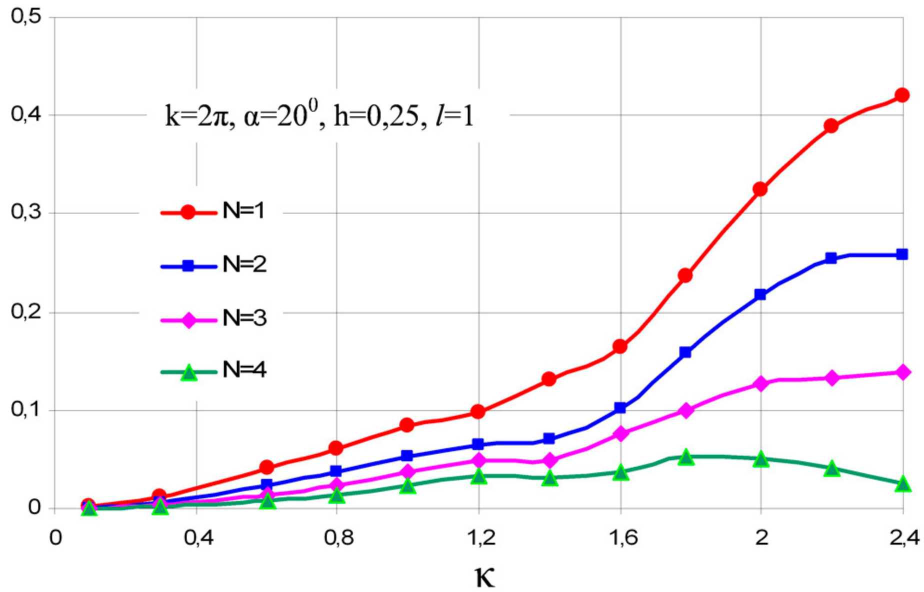

Fig.6.3. Dependence the absolute values of $C_1^{(N)}$ *as a function of* $\kappa = 2l/\lambda$ *for N order pre-Cantor grating.*

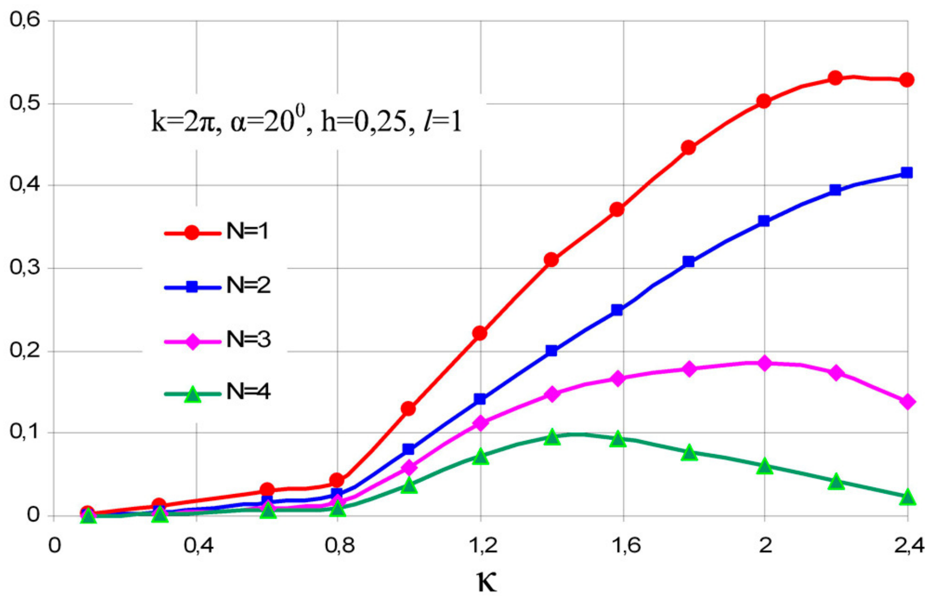

Fig. 6.4. Dependence the absolute values of $C_0^{(N)}$ *as a function of* $\kappa = 2l/\lambda$ *for N order pre-Cantor grating.*

In Fig. 6.5, 6.6, 6.7 absolute values of fields correspond to mark with numbers 1, 2, 3.

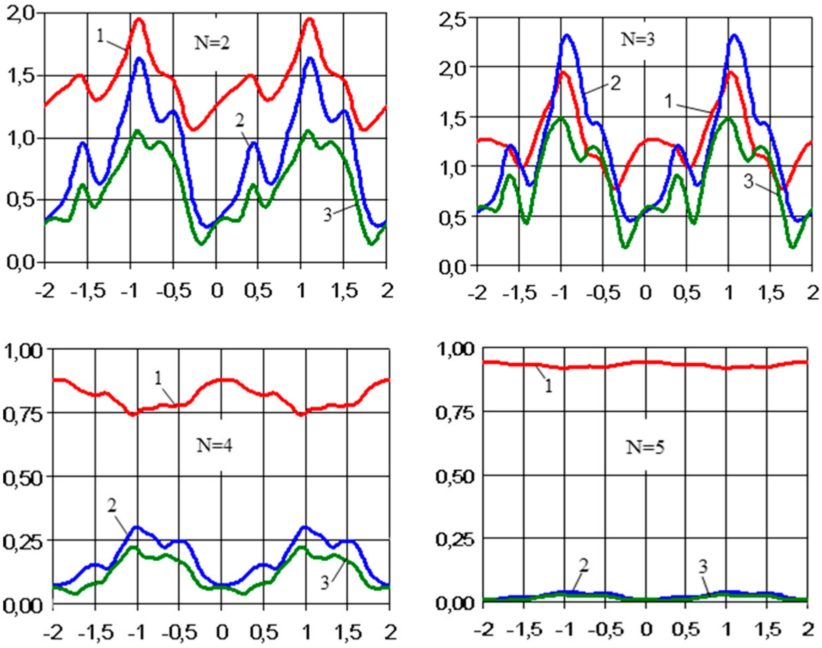

Fig.6.5. Dependence the absolute values of the field $H_x(y,z)$, $u_+^{(N)}(y,z)$, $u_-^{(N)}(y,z)$ in near zone from order pre-Cantor grating N=2,3,4,5, at k=2π, α=20^0, h=0.25, l=1.

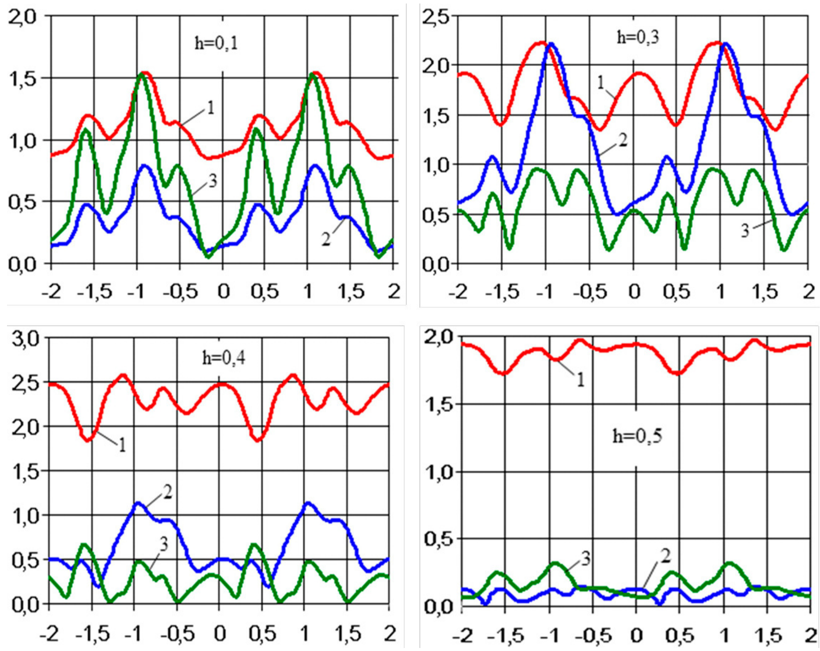

Fig.6.6. Dependence the absolute values of the field $H_x(y,z)$, $u_+^{(N)}(y,z)$, $u_-^{(N)}(y,z)$ in near zone from distance to shield h=0.1, h=0.3, h=0.4, h=0.5, at k=2π, α=20^0, N=3, l=1.

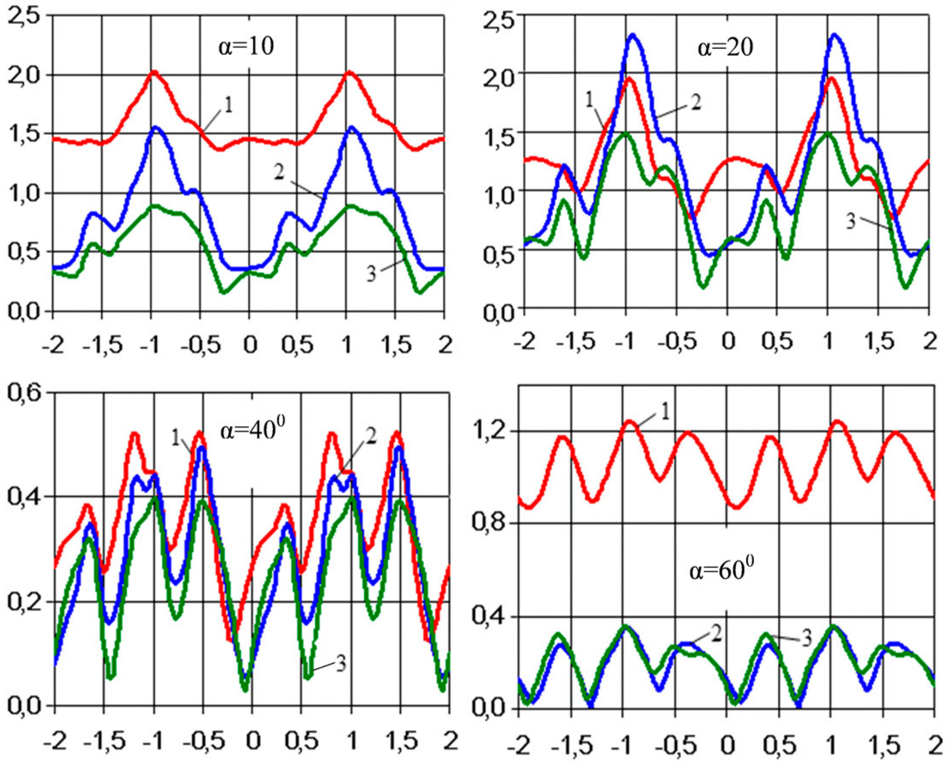

Fig.6.7. Dependence the absolute values of the field $H_x(y,z)$, $u_+^{(N)}(y,z)$, $u_-^{(N)}(y,z)$ in near zone from angel of incidence α=10^0, α=20^0, α=30^0 , α=40^0 at k=2π, N=3, h=0.25, l=1.

7. The conclusions of results and directions for further research

Singular integral equations for the boundary-value problem with Neumann parametric representation of the operator Hilbert have been obtained. Discrete mathematical model of the SIE with additional conditions by effective discrete singularities method had been constructed. Investigations of dependence the absolute values of complex amplitudes for the case of H polarization had been carried out. Plots of dependence the full, scattered and diffracted fields in the near zone of the order of the grating, distance to shield, angle of incidence for the case of H polarization had been constructed.

Future prospects in this field is the investigation of problems of the diffraction theory on periodic and bounded gratings with impedance boundary conditions (the Neumann and Dirichlet problems, cases E and H polarization), the construction of discrete mathematical models of these problems and carrying out numerical experiments based on them, the numerical solution using the DSM for periodic gratings lying on a dielectric shield. Next we will plan to do the numerical experiment of these problems based on hypersingular integral equations (HSIE) and compare the results for the case E polarization and for the case of H polarization.

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