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Analysis of the Buffer's Increment for the Billing System

A. Y. Pilipenko, V. F. Cherdyntseva
Institute of Mathematics, Ukrainian National Academy of Sciences, Ukraine
National Technical University of Ukraine "Kiev Polytechnic Institute", Ukraine

Billing system is one of the key systems of the telecommunication operator. The problem of incorrect calculation of subscribers for the provided services appears when billing system is overloaded. In other words, subscribers use different services free of charge. Purchase of a new billing system is expensive, so if the system is only partially satisfies all the requirements of the operator, the full replacement of its unprofitable. In this case, it is necessary to find other approaches that will minimize losses in upgrading the billing system. The purpose of the paper is to figure out the optimal value of the buffer increment of the billing system, and determine when invested capital would make a profit taking into account money discount factor, compared with the price of the costs for modernization.

Key words: *billing system, buffer, capacity, increment, modernization, on-line charging, Markov process.*

Білінгова система є однією з ключових систем в роботі операторів телекомунікаційних послуг. При навантаженій роботі білінгової системи виникає проблема, коли відбувається некоректний розрахунок абонентів за надані послуги. Іншими словами, абоненти використовують ті чи інші сервіси безкоштовно. Придбання нової білінгової системи вимагає великих фінансових витрат, тому, якщо система лише частково не задовольняє всіх вимог оператора, повна її заміна нерентабельна. У цьому випадку необхідно знайти інші підходи, які дозволять мінімізувати втрати при модернізації білінгової системи. Метою даної статті є визначити оптимальне значення прирощення ємності буфера очікування білінгової системи і моменту, коли інвестований капітал почне приносити прибуток з урахуванням коефіцієнта дисконтування грошей, порівняно з витратами на її модернізацію.

Ключові слова: *білінгова система, буфер, ємність, прирощення, модернізація, on-line тарифікація, марківський процес.*

Биллинговая система является одной из ключевых систем в работе операторов телекоммуникационных услуг. При нагруженной работе биллинговой системы возникает проблема, когда происходит некорректный расчет абонентов за предоставленные услуги. Иными словами, абоненты используют те или иные сервисы бесплатно. Приобретение новой биллинговой системы требует больших финансовых затрат, поэтому, если система лишь частично не удовлетворяет всех требований оператора, полная её замена нерентабельна. В этом случае необходимо найти другие подходы, которые позволят минимизировать потери при модернизации биллинговой системы. Целью данной статьи является определить оптимальное значение приращения емкости буфера ожидания биллинговой системы и момента, когда инвестированный капитал начнет приносить прибыль с учетом коэффициента дисконтирования денег, по сравнению с затратами на её модернизацию.

Ключевые слова: *биллинговая система, буфер, емкость, приращение, модернизация, on-line тарификация, марковский процесс.*

1. Introduction

Billing system is telecommunication software or hardware application that provides accounting the volume of provisioning services, charging in accordance with the tariffs of the company, and decrementing users' balance.

According to the analysis of literature and functioning of the billing system, one of the drawbacks was discovered. The problem of incorrect calculation of subscribers for the provided services appears when billing system is overloaded. That is subscribers may use different services free of charge. Since the telecommunication operators are commercial organizations, aimed at making a profit, then the correct operation of the billing system is one of the key issues.

Purchase of a new billing system is expensive. So, if the system only partially satisfies all the requirements of the operator, a complete replacement of its unprofitable. In this case, it is necessary to find other approaches that would minimize losses in upgrading the billing system.

One of the easy methods of solving the problem could be increasing the buffer size. However, the increase of buffer capacity entails monetary investment. Therefore it is necessary to figure out the optimal value of the buffer increment. And determine when invested capital would make a profit, compared with the price of the costs for modernization.

To calculate the profit in the future, we must take into account the discount factor – the interest rate used for the reappraisal of future income flows into a single value of the current cost.

2. Payment Methods and Principles of Mutual Settlements. Types of Billing

There are several ways to pay for telecommunications services:

- Credit;
- Imprest method;
- Debit.

Credit – the payment will be charged for a certain period for a service already provided. You pay for the right to use (monthly fee) or for the traffic (while using the communication channels).

At imprest method of mutual settlements a user pays a certain stipulated amount, and then pays the difference between the cost of rendered services and advance fee.

In debit methods of mutual settlements the operator receives advance payment, but the cost of the services provided could not exceed the amount of the deposit. Besides, the value of user fees and the charge for the provided services can be deducted from the deposit.

Credit principles of payments are called post-paid, and debit principles are called pre-paid.

In accordance with the credit payment methods, charging for provided services performed in two stages.

Duration of the connection, the amount of transmitted or received information, connection time, user attributes (direction, etc.) of a compound and type of service are fixed in a real-time on the first stage. These data are written in a standard recording CDR (Call Detail Record – is a data record produced by a telephone exchange or other telecommunications equipment documenting the details of a phone call that passed through the facility or device).

On the second stage, these data are collecting during some term, then they are charging on the basis of the individual tariff and summarizing for each user. This type of billing is called off-line (delayed billing).

In off-line billing of main and VAS services the operator must either reflect the kind of service in CDR, or simply fix the fact of use to collect a payment. While rendering some of the main and VAS services the amount of the credit provided for the user can be quite large, because of the difficulties in determining a debt on the account in advance. In this case we should use such type of billing, in which CDR entered into the system immediately after the end of providing the service. Usual, time of arrival varies from minutes to hours. This type of billing is called on-line billing. In this type of billing operator can control the user's debt in some limits, and even conduct pseudo-debit calculation.

Another type of billing that allows to control user's debt or debit balance in real-time – hot-line billing. CDR does not accumulate while using hot-line billing. Charging is conducted during the time of providing service. At the same time, system evaluates of both the cost and the ability to pay for a telecommunications service (debit principles of payments) or the difference between the maximum and current debt (for credit payments), followed by in-call decrementing of the balance. In hot-line billing the possibility to receive the services over a stipulated amount is excluded [1].

Thus, the billing can be classified by the kinds: off-line, on-line and hot-line. And by the types: post-paid and pre-paid.

3. Statement of the Problem

Pre-paid users, which are charging on-line or hot-line, can receive the service free of charge during the faulty operations of the billing system. This occurs when the buffer waiting is full, and the data switch of the billing system has no time to process the incoming packets. Then these packets receive requested services without fixing the provision of services and, accordingly, without charging.

3.1. Assumptions

Let A be a price of the buffer unit, C be the average packet price, n_0 the initial capacity of the buffer, λ the intensity of the arriving packets, μ the intensity of the processing arriving packets via a single device.

3.2. Problem

Find the optimum buffer size for the billing system to minimize losses during its modernization.

3.3. Solution

Assume, that the work of the billing system is described by Markov process. Let the system receives the independent flows of packets in a random way, of types $1,...,m$ with intensities $\lambda_1, \lambda_2, ...\lambda_m$, respectively.

Denote the packet price of the k-flow by C_k. Then the total intensity of packets arrival:

$$\lambda = \lambda_1 + \lambda_2 + ... + \lambda_m \tag{3.1}$$

The probability that arrived packet has k-th type equals:

$$p_k = \frac{л_k}{л} \quad (3.2)$$

Notice that the average price of incoming packet is:

$$C = \sum_k p_k C_k \quad (3.3)$$

Assume that incoming packets are processed with a single device with intensity μ.

Let $x(t)$ be a number of packets in the system at the moment t. Then $x(t)$ is described by the M/M/1/n$_0$ system, where n_0 is the initial capacity of the buffer.

It is well-known that the distribution of the system converges to a stationary distribution in a long time. Therefore we assume that $x(t)$ has a stationary distribution.

Let $\rho = \frac{\lambda}{\mu}$. Then the stationary distribution is equal to [2]:

$$\pi_k = \frac{\lambda^k}{\mu^k}\pi_0;$$

$$\sum_{k=0}^{n_0+1} \pi_k = 1; \ \frac{\lambda^k}{\mu^k} = \rho^k;$$

$$\pi_0 \sum_{k=0}^{n_0+1} \rho^k = 1; \quad (3.4)$$

$$\pi_0 = \frac{1-\rho}{1-\rho^{n_0+2}}; \ \pi_k = \frac{\rho^k(1-\rho)}{1-\rho^{n_0+2}}, \ k = \overline{0, n_0+1}.$$

<u>Remark</u>. If $\rho = 1$, then the stationary distribution $\pi_k = \frac{1}{n_0+2}$. This case will not be discussed.

If the system is in the state $n_0 + 1$, the new packet will be dropped. That is the probability of failure for the incoming packet is:

$$P_{failure} = \pi_{n_0+1} = \frac{\rho^{n_0+1}(1-\rho)}{1-\rho^{n_0+2}} \quad (3.5)$$

Denote the average number of lost packets during T by l, and the average loss for the time T by W. Then

$$l = \pi_{n+1} \cdot \lambda T \quad (3.6)$$

$$W = l \cdot \sum_k p_k C_k = \pi_{n_0+1} \lambda T \cdot C \quad (3.7)$$

Consider an infinitesimal period of time from t to $t + \Delta t$. Then the average number of losses from missed packets during the interval $[t, t+\Delta t]$ equals $\pi_{n_0+1}\lambda\Delta t$.

Thus, the rate of increase in losses from missed packet can be interpreted as

$$F_t = \pi_{n_0+1} \cdot \lambda C \quad (3.8)$$

Consider the time value of money to understand what is more appropriate for the operator: either to increase the buffer capacity of the billing system, thereby increasing the number of subscribers and profits, or to save money on upgrading the billing system and incur losses from packets lost.

Time value of money is a concept which is based on the assumption that the money should bring the percentage: the value of money today is higher than the value of the same amount received in the future. Everyone would prefer to get a certain amount of money today than the same amount in the future. Because, if you put some money in the bank, then next year you can get not only this amount of money, but also the interest on deposit. In other words, we can assume that $\$x$ at the moment t equivalent to $\$x\alpha^t$ at time 0, where the constant $\alpha \in (0;1)$ is called the discount factor [3].

This means that for recalculating of future income flows into a single value of the current cost we should use the integral effect, discounted to the present point of time, or integral discounted effect [3].

Thus, for an infinite time interval the average cost of lost packets, taking into account the discount factor is:

$$\int_0^\infty F_t \alpha^t dt = \int_0^\infty \pi_{n_0+1} \lambda C \alpha^t dt = \frac{\pi_{n_0+1}\lambda C}{\ln \frac{1}{\alpha}} \tag{3.9}$$

The increase in total income from increasing the size of buffer in a given instant is equal to:

$$f(n) = \frac{\pi_{n+1}\lambda C}{\ln \frac{1}{\alpha}} + \frac{\pi_{n_0+1}\lambda C}{\ln \frac{1}{\alpha}} - A(n - n_0) \tag{3.10}$$

Where A is a price of the buffer unit, $C = \sum_k p_k C_k$ is the average packet price, n_0 the initial capacity of the buffer, n is the buffer size after expansion.

To find the largest profit from the increasing of buffer explore this function on maximum.

To simplify the calculations introduce the notation $B = \frac{\rho(1-\rho)\lambda C}{\ln {}^1\!/\!_\alpha}$, then:

$$f(n) = -\frac{\rho^n}{1-\rho^{n+2}} \cdot B + \frac{\rho^{n_0}}{1-\rho^{n_0+2}} \cdot B - A(n - n_0) \to \max \tag{3.11}$$

Transform this function to a function that depends on the variable x:

$$f(x) = -\frac{\rho^x}{1-\rho^{x+2}} \cdot B + \frac{\rho^{n_0}}{1-\rho^{n_0+2}} \cdot B - A(x - n_0) \tag{3.12}$$

To find the maximum of the function $f(x)$ explore its derivative:

$$f'(x) = \left(-\frac{\rho^x}{1-\rho^{x+2}} \cdot B + \frac{\rho^{n_0}}{1-\rho^{n_0+2}} \cdot B - A(x - n_0) \right)' \tag{3.13}$$

$$f'(x) = -\frac{B[\rho^x \cdot \ln\rho \cdot (1-\rho^{x+2}) + \rho^{x+2} \cdot \ln\rho \cdot \rho^x]}{(1-\rho^{x+2})^2} - A = \\ = -\frac{\rho^x \cdot \ln\rho}{(1-\rho^{x+2})^2} \cdot B - A = -\frac{\rho^x \cdot \ln\rho \cdot B + A(1-\rho^{x+2})^2}{(1-\rho^{x+2})^2} \quad (3.14)$$

Calculating the second derivative and noting that $B > 0$ when $\rho \in (0;1)$, $B < 0$ when $\rho > 1$, it is easy to check that $f''(x) < 0$ for $x > -2$. Therefore, the function f is strictly convex.

Since $\lim_{x\to -2+0} f(x) = \lim_{x\to +\infty} f(x) = -\infty$, then f has a unique maximum on the interval $(-2;+\infty)$. To find it we solve the equation $f'(x) = 0$. So the derivative is equal to 0 if:

$$-\rho^x \cdot \ln\rho \cdot B - A(1-\rho^{x+2})^2 = 0 \quad (3.15)$$

Denote $z = \rho^x$. Thus

$$-z \cdot \ln\rho \cdot B - A(1 - z\rho^2)^2 = 0 \quad (3.16)$$

$$A\rho^4 \cdot z^2 + z(\ln\rho \cdot B - 2A\rho^2) + A = 0 \quad (3.17)$$

This equation is quadratic relative to z. The discriminant of this equation is:

$$D = \ln\rho \cdot B(\ln\rho \cdot B - 4A\rho^2) > 0 \quad (3.18)$$

Since

$$\ln\rho \cdot B = \ln\rho \cdot \frac{\rho(1-\rho)\lambda C}{\ln\frac{1}{\alpha}} < 0, \quad (3.19)$$

the roots of a quadratic equation are:

$$z_{\pm} = \left[\begin{array}{l} \dfrac{-(\ln\rho \cdot B - 2A\rho^2) + \sqrt{\ln\rho \cdot B(\ln\rho \cdot B - 4A\rho^2)}}{2A\rho^4}, \\ \dfrac{-(\ln\rho \cdot B - 2A\rho^2) - \sqrt{\ln\rho \cdot B(\ln\rho \cdot B - 4A\rho^2)}}{2A\rho^4}. \end{array}\right. \quad (3.20)$$

Therefore, the optimal value of x is:

$$x_+^* = \log_\rho z_+ = \frac{\ln z_+}{\ln\rho} \text{ or } x_-^* = \log_\rho z_- = \frac{\ln z_-}{\ln\rho} \quad (3.21)$$

As mentioned above, f attains a unique maximum on the interval $x \in (-2;+\infty)$. Therefore, the equation $f'(x) = 0, x \in (-2;+\infty)$ has exactly one root. So

$$x^* = \max\{x_+^*, x_-^*\} = \begin{cases} \dfrac{\ln z_+}{\ln\rho}, \text{ if } \rho \in (1;+\infty), \\ \dfrac{\ln z_-}{\ln\rho}, \text{ if } \rho \in (0;1). \end{cases} \quad (3.22)$$

This means that if $x^* \le n_0$, increasing the buffer waiting is unprofitable. If $x^* > n_0$, then increasing the size of buffer waiting would minimize the loss of telecommunications operator and allow it to obtain the greater profit.

Remark. If the cost of the work for replacement of equipment is $C > 0$, then the profit function would be:

$$g(x) = -\frac{\rho^x}{1-\rho^{x+2}} \cdot B + \frac{\rho^{n_0}}{1-\rho^{n_0+2}} \cdot B - A(x-n_0) - C \tag{3.23}$$

Note that $g'(x) = f'(x)$. Therefore, these functions have the same maximum at x^*.

Then, if $g(x^*) > 0$ it is necessary to increase the buffer waiting for the billing system to obtain greater profit. Otherwise, if $g(x^*) > 0$ the increasing of the buffer capacity is impractical.

Conclusion

1. Proposed method for calculating the capacity of the billing system, allows to associate the economic component of the telecommunications network operator with the technical parameters.
2. Using a new method we can calculate the optimum value of the increment of the buffer waiting and determine at what moment committed facilities will make a profit, compared with the price of the costs for modernization.
3. The optimal buffer waiting size allows the operator to get the greater profit and minimize losses when upgrading the billing system.

REFERENCES

1. Mussel K.M.: Provision and billing of communication services. System Integration. Moscow, Eco-Trendz, 2003.
2. Gnedenko B.V., Kovalenko I.N.: Queueing Theory. Moscow, High School, 1982.
3. Bellman R.: Dynamic Programming, ed. Vorobyov N.N. Moscow, Publishing House of Foreign Literature, 1960.
4. Ladyk O.I., Tsema V.F.: Analytical modeling billing system with random parameters. Scientific Notes of the Ukrainian Research Institute of Communications, № 1 (13), pages 27–30, Kiev, Ukraine, 2010.
5. Пилипенко А. Ю., Чердынцева В.Ф. Метод уменьшения нагрузки на биллинговую систему в режиме критического состояния // СВЧ-техника и телекоммуникационные технологи: матер. 22-ой межд. науч.-техн. конф. КрыМиКо–2012, Севастополь, 2012 г. – Севастополь, 2012, ст. 403-404.

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