Вісник Харківського національного університету
Серія «Математичне моделювання. Інформаційні технології. Автоматизовані системи управління»
УДК 510.643:519.68 № 733, 2006, с.149-159

On the Decidability of Propositional Metric Temporal Calculus PTC(MT)

Natalya Keberle
Dept. of Information Technologies, Zaporozhye National University†

The paper introduces propositional metric temporal calculus *PTC(MT)* dealing with metric properties of time – transitivity and distance between time points. The paper proves that *PTC(MT)* is decidable.

1. Introduction

Temporal logic is a kind of symbolic modal logic [1] dealing with domain description statements, which are interpreted over the time flow, either point-based or interval-based. First introduced by Prior in 1957, temporal (or tense) logics relate tenses and modalities, and provide a basis for description of the semantics of the evolving world.

The reviews of known logical systems involving temporal modalities can be found in [2-4]. Among these systems are Lemmon's minimal system K_t (with the unary operators F – "somewhere in the future", G – "always in the future" and their mirrors), von Wright system "And then" (the binary operator T_w, and basic construct $pT_w\, q$ – "p and then q"), Scott's system "And next instant" (the unary operator T_s, basic construct $T_s\, p$ – "in the next time point will be p"), logical system with Kamp's binary temporal modalities U – "until" and S – "since" [5].

Temporal aspect is also of great interest for hybrid logics, where it is possible to directly refer to worlds/times/states in logical formulae. E.g. Rescher's chronological calculus [6] introduces the operator of chronological realization, which binds an event to the particular real date/time.

Temporal logics are widely accepted languages for specifying properties of reactive systems and their behaviour over time [7-8], and for the description of concurrent object-based systems: process controls, fault tolerant systems, distributed AI [9]. Its application to the description of evolving behaviour of dynamic domains is under detailed investigation, particularly for the purposes of knowledge representation on the Semantic Web (see e.g. [10-11]).

The examples of propositional temporal logics for linear time are *LTL* [12], *PTL* [13], *Timed PTL* [8], and the set of Propositional Linear Temporal Logics from [4].

Metric temporal logic with modalities *Fn* ("it will be the case after n time points") and *Pn* ("it was the case n time points ago") allows in addition to description of precedence of events to explicitly state distances (in time points) between the occurrences of events.

This logic is positioned between non-metrical temporal logics and hybrid logics.

† kenga@zsu.zp.ua

Indeed, from the one point, constructs like $Fnp \vee Fmq$ tell that *p* will be true in *n* time moments or *q* will be true in *m* time moments, thus one can use primitive arithmetic operations to calculate the difference between time moments when the *p* or *q* occur. At the same time, non-metrical modal operators of other systems (like *F*, *G*, T_s, *U* and *S*) can be easily presented via metric one (see e.g. [1]). From the other point, it is impossible to set or get absolute values of time moments when *p* or *q* occur.

However, the review of other researches in temporal logics has showed that the complete propositional metric temporal calculus with temporal modalities *Fn* and *Pn*, as they were introduced in [5], was not investigated with respect to the logical properties of an arbitrary formal system: completeness, soundness and decidability.

The aim of the paper is to introduce propositional metric temporal calculus *PTC(MT)*, and to prove decidability of *PTC(MT)*. The work on *PTC(MT)* , particularly soundness and completeness analysis, was presented in [15].

The paper is structured as follows: Section 2 introduces the *PTC(MT)*; Section 3 describes the tableau procedure for checking formula satisfiability; Section 4 analyses decidability of *PTC(MT)*; Section 5 concludes the paper.

2 *PTC(MT)*

Propositional metric temporal calculus considers time having linear discrete structure, infinite into the past and to the future, assumes that time points are organized with reflexive and transitive ordering relation.

Such structure of time is isomorphic to the structure $\langle \mathrm{Z}, < \rangle$, where Z - is a set of integers, and < - is a strict ordering relation.

Formal system is defined if defined are alphabet, rules of formulae construction, the set of axioms, and the set of the deduction rules.

2.1 Alphabet and formulae construction rules

The alphabet of *PTC(MT)* consists of:

(a) Propositional variables $p, q, r, s, ...$;

(b) Primitive propositional connectives $\neg, \supset$, and additional connectives $\wedge, \vee, \equiv$, defined over primitive ones in the usual way;

(c) Temporal operators *Fn*, *Pn* (*Fn* –« it will be the case after *n* time points», *Pn* – «it was the case *n* time points ago»);

PTC(MT) terms are:

(a) $\nu, \nu_1, \nu_2, ...$ are natural numbers and «0»;

(b) $i, i_1, ..., j, j_1, ...$ are numerical variables;

(c) if $n_1, ..., n_m$ are natural numbers and «0» or numerical variables, and θ – *m*-ary operator, then $\theta(n_1, ..., n_m)$ – is a term.

Formulae are constructed following the rules:

(a) Every propositional variable is a formula;

(b) If φ and ψ are formulae, then $\neg\varphi$, $\varphi \supset \psi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \equiv \psi$ are also formulae;

(c) If $\Pr^m$ is a predicate letter denoting *m*-ary predicate, defined over integers (e.g., «=», «>»,…), and $n_1, ..., n_m$ – are terms, then $\Pr^m(n_1, ..., n_m)$ - is a formula;

(d) If φ – is a formula, then $Fn\varphi$, $Pn\varphi$, $\exists i\varphi$, $\forall i\varphi$ – are also formulae.

Alphabet of *PTC(MT)* is defined.

Definition 1.

Numerical variable *i* occurs free in a formula φ, if it is not within the scope of any quantifier in φ.

Definition 2.

Term *n* is free in a formula φ for a numerical variable *j*, if there are no free occurrences of j in φ, such that *j* is within the scope of any quantifier $\forall i_m$, where i_m is a numerical variable in the term *n*.

2.2 Axioms and deduction rules

PTC(MT) axioms set consists of all axioms of the propositional calculus and some axioms of temporal logic, taken from [1-3].

Following formulae are axioms (propositional axioms are correspondent to L4 system, see [14, p.49]):

(A1) $p \supset (q \supset p)$;

(A2) $(p \supset (q \supset r)) \supset ((p \supset q) \supset (p \supset r))$;

(A3) $p \wedge q \supset p$

(A4) $p \wedge q \supset q$

(A5) $p \supset (p \vee q)$

(A6) $q \supset (p \vee q)$

(A7) $p \supset (q \supset (p \wedge q))$

(A8) $(p \supset q) \supset ((r \supset q) \supset ((p \vee r) \supset q)$

(A9) $(p \supset q) \supset ((p \supset \neg q) \supset \neg p)$

(A10) $\neg\neg p \supset p$

(AMT1) $(\neg Fn \neg (p \supset q)) \supset (Fnp \supset Fnq)$ – logical homogeneity in the future

(AMT1.1) $(\neg Pn \neg (p \supset q)) \supset (Pnp \supset Pnq)$ – logical homogeneity in the past

(AMT2) $Fn \neg Pn \neg p \supset p$

(AMT2.1) $Pn \neg Fn \neg p \supset p$

(AMT3) $Fm \exists iFip \supset \exists iFmFip$

(AMT3.1) $Pm \exists iPip \supset \exists iPmPip$

(AMT4) $Fm \exists iPip \supset \exists iFmPip$

(AMT4.1) $Pm \exists iFip \supset \exists iPmFip$

(AMT5) $F(m+n)p \supset FmFnp$

(AMT5.1) $P(m+n)p \supset PmPnp$

(AMT6) $\neg Fnp \supset Fn \neg p$ – infinity into the future

(AMT6.1) $\neg Pnp \supset Pn \neg p$ – infinity into the past

(AMT7) $Fn \neg p \supset \neg Fnp$ – nonbranching in the future

(AMT7.1) $Pn \neg p \supset \neg Pnp$ – nonbranching in the past

(AMT8) $FmFnp \supset F(m+n)p$ – transitivity in the future

(AMT8.1) $PmPnp \supset P(m+n)p$ – transitivity in the past

(AMT9) $(m = n+k) \supset (FmPnp \supset Fkp)$ – iteration of temporal modalities

Propositional axioms are independent with respect to *PTC(MT)*, the same applies for temporal axioms.

Deduction rules for calculus *PTC(MT)* are:

(R1) $\dfrac{\varphi, \varphi \supset \psi}{\psi}$ – Modus Ponens

(R2) $\dfrac{\varphi(p)}{\psi(p/\gamma)}$ – substitution rule (ψ is obtained after replacing in φ all occurrences of a propositional variable p with formula γ)

(R3) $\dfrac{\varphi}{\neg Fn \neg \varphi}$ – the rule of deriving "always in the future"

(R4) $\dfrac{\varphi}{\neg Pn \neg \varphi}$ – the rule of deriving "always in the past"

Let φ be a *PTC(MT)* formula that does not contain numerical variable *i*, $\varphi[j/i]$ be a *PTC(MT)* formula with all free occurrences of a numerical variable *j* replaced with *i*. Then the following deduction rule may be applied:

(R5) $\dfrac{\varphi[j/i]}{\forall i \varphi}$ – the generalization rule

If φ is a *PTC(MT)* formula which contains numerical variable *i*, and $\varphi[i/n]$ be a *PTC(MT)* formula with all occurrences of a numerical variable *i* replaced with term *n*, which is free for *i* in φ, then the following deduction rule may be applied:

(R6) $\dfrac{\forall i \varphi}{\varphi[i/n]}$

Calculus is constructed.

Throughout this paper we restrict the discussion with binary operations "+", "–" for *PTC(MT)* terms construction and use the only binary predicate "="("equality").

Definition 3.

Formula φ is called **atomic**, if φ is either a propositional variable or its negation, or a formula of the view $\Pr^m(n_1,...,n_m)$ or its negation.

Definition 4.

Formula φ is in **negation normal form** (n.n.f.), if for every subformula $\neg\psi$ formula ψ is atomic, and the whole formula φ is constructed without binary propositional connectives $\supset, \equiv$.

Theorem 1.

Let φ be a formula from *PTC(MT)*.

Then $\vdash \varphi \equiv \psi$, where ψ - is a formula in negation normal form (n.n.f.).

The proof of this fact is shown in the [15].

Definition 5.

Formula φ is in ***FnPn*-normal form** (*FnPn*-n.f.), if it can be presented as:

$$\varphi \equiv \bigvee_{k=1}^{N} \Big(\bigwedge_{r^1=0}^{N_k^1} F\nu_{kr^1}\varphi_{kr^1} \wedge \bigwedge_{r^2=0}^{N_k^2} P\nu_{kr^2}\varphi_{kr^2} \wedge$$

$$\wedge \bigwedge_{r^3=0}^{N_k^3} \exists i_{kr^3} \Pr^{s+1}(i_{kr^3}, \nu_1, \ldots \nu_s) \wedge \bigwedge_{r^4=0}^{N_k^4} \forall i_{kr^4} \Pr^{s+1}(i_{kr^4}, \nu_1, \ldots \nu_s) \wedge$$

$$\wedge \bigwedge_{\substack{\overline{\alpha,\beta} \\ r^j = \overline{0, N_k^j} \\ d = \overline{0, D_k^j}}} \alpha\, \nu_{kr^j}\, \beta\, i_{kr^j 1}\, \alpha\, i_{kr^j 1} \ldots \beta\, i_{kr^j d}\, \alpha\, i_{kr^j d}\, \varphi_{kr^j} \Big)$$

where

- N - is a number of disjuncts in a formula,
- k - is an internal index for referencing disjuncts within the formula,
- $N_k^j \geq 0$ - is a number of conjuncts of a particular conjunct form within *k*-th disjunct
- $j = \overline{1,\ldots,2^{2 \cdot D_k}}$ - is an index of a particular conjunct form within *k*-th disjunct,
- $\alpha \in \{F, P\}$ - is a symbol, partially denoting one of temporal modalities,
- $\beta \in \{\exists, \forall\}$ - is a symbol denoting one of quantifiers,
- $r^j = \overline{0,\ldots,N_k^j}$ - is an internal index for referencing formulae of a particular conjunct form within *k*-th disjunct,
- $d = \overline{1,\ldots,D_k^j}$ - is an internal index for referencing elements of the form $\beta_{kr^j}\ i_{kr^j}\ \alpha_{kr^j} i_{kr^j}$ within a formula in the r^j-th conjunct of the particular conjunct form within *k*-th disjunct,
- $D_k^j \leq D_k$ - is the number of quantifiers in the particular conjunct form within *k*-th disjunct,
- D_k - is the maximal number of quantifiers among all particular conjunct forms within *k*-th disjunct,
- φ_{kr^j} - are atomic formulae.

FnPn-n.f. of a *PTC(MT)* formula is a list of alternative histories of states of some object from a domain.

Theorem 2.

Let φ be a formula of *PTC(MT)* in n.n.f. Then $\vdash \varphi \equiv \psi$, where ψ is a formula in *FnPn*-normal form. The proof of this fact is shown in the [15].

3 Tableau procedure for checking PTC(MT) formula satisfiability

Construct a model of an arbitrary *PTC(MT)* formula. It is a widely accepted technique [7-8, 10-11, 16] to use tableau rules to construct a model for a modal system.

Definition 6.

Let φ be a formula in *FnPn*-n.f., and ψ be a subformula of φ. A sequence of formulae lists $< \zeta_0, \zeta_1, \ldots, \zeta_m, \zeta_{-1}, \ldots \zeta_{-s} >$, linearly ordered with a binary relation R (reflexive and transitive), forms a **chain** Z_{φ} for the formula φ, if this sequence is constructed following the set of rules, presented in the Table 1.

Table 1. Rules for construction of a semantic tableau for checking PTC(MT) formula satisfiability.

(0-rule)	Condition:	$\psi = \varphi$
	Action:	$\zeta_0 = \psi$
($\wedge$-rule)	Condition:	1. $\psi = \psi_1 \wedge \psi_2$ 2. $\{\psi_1, \psi_2\} \cap \zeta = \varnothing$
	Action:	$\zeta = \zeta \cup \{\psi_1, \psi_2\}$
($\vee$-rule)	Condition:	1. $\psi = \psi_1 \vee \psi_2$ 2. $\{\psi_1, \psi_2\} \cap \zeta = \varnothing$
	Action:	Either $\zeta = \zeta \cup \{\psi_1\}$ or $\zeta = \zeta \cup \{\psi_2\}$
($F\nu$-rule)	Condition:	1. $\psi = F\nu\psi_1$ 2. $\psi \in \zeta_k$ 3. $\nu \geq 1$
	Action:	1. If there is no $\zeta_{k+1} : \zeta_{k+1} \in \mathrm{Z}_{\varphi}$, then such list is created and new formula $\psi' = F(\nu - 1)\psi_1$ is added to the ζ_{k+1}, $\psi' \in \zeta_{k+1}$ 2. If exists $\zeta_{k+1} : \zeta_{k+1} \in \mathrm{Z}_{\varphi}$, then $\psi' = F(\nu - 1)\psi_1$ is added to the ζ_{k+1}, $\psi' \in \zeta_{k+1}$ 3. Between ζ_k and ζ_{k+1} relation $R(\zeta_k, \zeta_{k+1})$ is set.
($P\nu$-rule)	Condition:	1. $\psi = P\nu\psi_1$ 2. $\psi \in \zeta_k$ 3. $\nu \geq 1$
	Action:	1. If there is no $\zeta_{k-1} : \zeta_{k-1} \in \mathrm{Z}_{\varphi}$, then such list is created and new formula $\psi' = P(\nu - 1)\psi_1$ is added to the ζ_{k-1}, $\psi' \in \zeta_{k-1}$ 2. If exists $\zeta_{k-1} : \zeta_{k-1} \in \mathrm{Z}_{\varphi}$, then $\psi' = P(\nu - 1)\psi_1$ is added to the ζ_{k-1}, $\psi' \in \zeta_{k-1}$ 3. Between ζ_k and ζ_{k-1} relation $R(\zeta_k, \zeta_{k-1})$ is set.
$\exists iFi$-rule	Condition	1. $\psi = \exists iFi\psi_1$ 2. $\psi \in \zeta_k, \psi_1 \notin \zeta_k$

	Action	Either $\psi_1 \in \zeta_k$ or new formula $\psi' = F1\,\exists iFi\psi_1$ belongs to ζ_k, $\psi' \in \zeta_k$
$\exists iPi$ -rule	Condition	1. $\psi = \exists iPi\psi_1$ 2. $\psi \in \zeta_k, \psi_1 \notin \zeta_k$
	Action	Either $\psi_1 \in \zeta_k$ or new formula $\psi' = P1\,\exists iPi\psi_1$ belongs to ζ_k, $\psi' \in \zeta_k$
$\forall iFi$ -rule	Condition	1. $\psi = \forall iFi\psi_1$ 2. $\psi \in \zeta_k, \psi_1 \notin \zeta_k$
	Action	1. $\psi_1 \in \zeta_k$ 2. For each $\zeta_j : \zeta_j \in Z_\varphi, j > k$, such that the relation $R(\zeta_k, \zeta_j)$ is set, $\psi \in \zeta_j$
$\forall iPi$ -rule	Condition	1. $\psi = \forall iPi\psi_1$ 2. $\psi \in \zeta_k, \psi_1 \notin \zeta_k$
	Action	1. $\psi_1 \in \zeta_k$ 2. For each $\zeta_j : \zeta_j \in Z_\varphi, j < k$, such that the relation $R(\zeta_k, \zeta_j)$ is set, $\psi \in \zeta_j$
$\exists i \Pr^2(i, \theta(\nu_1, \nu_2))$ -rule (for predicate letter "=")	Condition	1. $\psi = \exists i \Pr^2(i, \theta(\nu_1, \nu_2))$ 2. $\psi \in \zeta_k$
	Action	If there is no $\zeta_i \in Z_\varphi$, such that $i = \theta(\nu_1, \nu_2)$, then such list is created.
$\forall i \Pr^2(i, \theta(\nu_1, \nu_2))$ -rule (for predicate letter "=")	Condition	1. $\psi = \forall i \Pr^2(i, \theta(\nu_1, \nu_2))$ 2. $\psi \in \zeta_k$
	Action	If there is no $\zeta_i \in Z_\varphi$, such that $i = \theta(\nu_1, \nu_2)$, then such list is created.

Table 1 does not contain rules for resolving formulae like $\psi = Fi\psi_1$, where i is a numerical variable, or like $\psi = \forall i\psi_1$. Such formulae can be presented in the form $\forall jFj\psi_1$ with application of the deduction rules R5, R6.

It also should be pointed out that the $\forall iFi$- and $\forall iPi$-rules reflect the transitivity and reflexivity of the relation R between possible worlds at different time points. According to the definition of a model for a modal system (see [16]) the model of the propositional metric temporal calculus *PTC(MT)*, constructed according to the rules from Table 1, is S4-model.

Definition 7.

A set $\{Z_\varphi^1,...,Z_\varphi^k\}$ of chains constructed according to the rules enlisted in the Table 1, is called a **construction** C_φ.

Definition 8.

Chain Z_φ is **closed**, if it contains a formulae list ζ such that for some propositional variable *p* both p and $\neg p$ are in ζ. Construction C_φ is **closed** if all chains in it are closed.

Given a *PTC(MT)* formula φ, its construction creation procedure can be described as follows. Construction creation starts from applying 0-rule, then apply $\vee$-rule until there will not be any unresolved subformulae ψ having disjunction, then apply $\wedge$-rule until there will not be any unresolved subformulae ψ having conjunction. After that apply $\exists i \Pr^2(i,\theta(\nu_1,\nu_2))$- and $\forall i \Pr^2(i,\theta(\nu_1,\nu_2))$-rules, which will introduce new (though empty) formulae lists, then apply $F\nu$- and $P\nu$-rules until there will not be any unresolved subformulae of that form. Finally, apply $\exists iFi$-, $\exists iPi$-rule and then $\forall iFi$- and $\forall iPi$-rules. This process will be continued until for each chain there will be a formulae list, which fulfills one of the following two conditions: either this chain is closed, or this chain with the same set of formulae is already in the construction.

Definition 9.

Construction C_φ is **complete** if no tableau rule is applicable to it.

Definition 10.

Let φ be a formula of *PTC(MT)*. A **model for** φ will be any chain Z_φ, which is not closed.

Definition 11.

Formula φ is **satisfiable** if and only if φ has a model defined over the construction C_φ.

Definition 12.

Formula φ is **logically valid** (denoted as $\models$) if and only if $\neg\varphi$ does not have a model defined over the construction $C_{\neg\varphi}$ (in other words, $\neg\varphi$ is **unsatisfiable**).

Metatheorem 1.

PTC(MT) is sound.

The proof of this fact is shown in the [15].

Metatheorem 2.

$\vdash \varphi$ iff $\models \varphi$ (completeness of *PTC(MT)*)

The proof of this fact is also shown in the [15].

4 *PTC(MT)* decidability

A formal theory is decidable if there is an effective decision procedure of checking whether a given formula is satisfiable.

Let $|C_\varphi|$ be cardinality of a construction C_φ of a formula φ – the number of chains Z_φ for the formula φ. Let $|Z_\varphi|$ be cardinality of a chain Z_φ – the number of formulae lists ζ_i in the chain. Finally, let $|\zeta|$ be cardinality of a formulae list ζ from the chain Z_φ of the construction C_φ of formula φ – the number of formulae in the formulae list ζ.

Lemma 1.

For arbitrary *PTC(MT)* formula φ the process of completing a construction C_φ always terminates after finitely many steps.

Proof: according to the Theorem 2 without loss of generality let φ be in *FnPn*-n.f. Construction C_φ is finite if and only if it consists of finite set of chains Z_φ, each chain Z_φ is also finite, i.e. consists of finite set of formulae lists ζ_i, and each formulae list ζ_i also consists of finite set of subformulae of the formula φ.

The analysis of the rules from Table 1 shows that

$\forall Z_\varphi \in C_\varphi \;\; |Z_\varphi| := |Z_\varphi| + 1$ in case of application of $F\nu$-, $P\nu$-, $\exists iFi$-, $\exists iPi$,

$\exists i \Pr^2(i, \theta(\nu_1, \nu_2))$-, $\forall i \Pr^2(i, \theta(\nu_1, \nu_2))$- rules, and remains the same otherwise.

Consider a chain Z_φ and evaluate the number of formulae in a given formulae list:

$\forall \zeta_i \in Z_\varphi \;\; |\zeta_i| := |\zeta_i| + 1$ in case of application of $F\nu$-, $P\nu$-, $\exists iFi$-, $\exists iPi$-, $\forall iFi$-, $\forall iPi$-, $\vee$- rules,

$\forall \zeta_i \in Z_\varphi \;\; |\zeta_i| := |\zeta_i| + 2$ in case of application of $\wedge$- rule, and remains the same otherwise.

Recall that there are no more than $2^{2 \cdot D_k}$ different conjunct forms in the *k*-th disjunct in φ, and no more than N_k^j conjuncts within each conjunct form. The cardinality of the initial formulae list, ζ_0, for the formula φ is bounded:

$$|\zeta_0| \leq 2^{N_k^1 + N_k^2 + \ldots + N_k^{2^{2 \cdot D_k}}} - 1$$

It is obvious, that $|\zeta_i| \leq |\zeta_0|$ for any $\zeta_i \in Z_\varphi$, as far as only in ζ_0 will be conjuncts of the forms $\exists i_{kr^3} \Pr^{s+1}(i_{kr^3}, \nu_1, \ldots \nu_s)$, $\forall i_{kr^3} \Pr^{s+1}(i_{kr^3}, \nu_1, \ldots \nu_s)$.

The cardinality of the chain, corresponding to the whole formula φ is also bounded:

$$\left|Z_\varphi\right| \le \max\{v : Fv\varphi_{kr^j} \in Sub(\varphi), Fv\beta\, i_{kr^j{}_1}\, \alpha\, i_{kr^j{}_1} ...\beta\, i_{kr^j{}_d}\, \alpha\, i_{kr^j{}_d} \varphi_{kr^j} \in Sub(\varphi)\} +$$
$$+ \max\{v : Pv\varphi_{kr^j} \in Sub(\varphi), Pv\beta\, i_{kr^j{}_1}\, \alpha\, i_{kr^j{}_1} ...\beta\, i_{kr^j{}_d}\, \alpha\, i_{kr^j{}_d} \varphi_{kr^j} \in Sub(\varphi)\}$$

where:

- φ_{kr^j} - are atomic formulae within φ,

- $Sub(\varphi)$ - is the set of all subformulae of φ.

According to the definition of a chain and a construction, one may observe that one chain corresponds to one subformula φ.

Denote $A_k^j = 1 + N\beta_k^j + N\alpha_k^j$ - the number of subformulae, constructed for *j*-th particular conjunct form in *k*-th disjunct of φ. Here $N\beta_k^j = D_k^j$ - is the number of quantifiers within the particular conjunct form (generally, $N\beta_k^j$ can also be equal to 0, for example for the *1*-th and the *2*-nd conjunct forms of *FnPn*-n.f.), $N\alpha_k^j$ - is the number of temporal modalities within the particular conjunct form in *k*-th disjunct of φ ($N\alpha_k^j$ can be equal to zero, e.g. for the *3*-rd and the *4*-th conjunct forms of *FnPn*-n.f.).

Now it is possible to evaluate $M_k = \sum_{j=1}^{2^{2\cdot D_k}} A_k^j \cdot N_k^j$ - the general quantity of subformulae across all conjuncts in *k*-th disjunct of φ (again, φ is assumed to be in *FnPn*-n.f.).

Then the cardinality of the set $Sub(\varphi)$, and, consequently, of the construction C_φ for the formula φ can be restricted as $\left|C_\varphi\right| \le 2^{\sum_{k=1}^{N} M_k} - 1$, i.e. it is finite. End of proof.

Metatheorem 3.
PTC(MT) is decidable.
The proof of this fact is based on Lemma 1.

5 Conclusions

The paper introduces propositional metric temporal calculus *PTC(MT)*. The work on *PTC(MT)* logical analysis, particularly on soundness and completeness, was presented in [15]. The paper proves that *PTC(MT)* with temporal modalities *Fn* and *Pn* is decidable.

The work will be continued in the following direction: all results obtained for the propositional metric temporal system *PTC(MT)* will be considered for the Description Logics family, which are de facto standard for presentation of ontologies on the Semantic Web.

REFERENCES

1. Clifford J.E. Tense and Tense Logic. Mouton Publishers, The Hague, The Netherlands, – 1975. – 173 pp.
2. Ishmuratov A.T. Logical theories of temporal contexts (temporal logic). Kyiv: Naukova dumka, – 1981. – 150 pp. (in Russian)
3. Karavaev E.F. Foundations of temporal logic. Leningrad: Leningrad University, – 1983. – 177 pp. (in Russian)
4. Sistla A.P., Clarke E.M. The Complexity of Propositional Linear Temporal Logics. J.ACM, Vol.32, No.3, July 1985, pp.733-749.
5. Prior A. Past, Present and Future. Oxford: Oxford University Press, – 1967.
6. Rescher N., Urquhart A. Temporal Logic. New York: Springer-Verlag, – 1971.
7. Manna Z., Pnueli A. The temporal logic of Reactive and Concurrent Systems: Specification. Springer-Verlag, New York, 1992.
8. Alur R., Henzinger T.A. A Really Temporal Logic. J. ACM, Vol. 41, No.1, January, 1994, pp.181-204.
9. Fisher M. A Survey of Concurrent METATEM – the Language and its Applications. In: Proc. 1st Intl. Conf. on Temporal Logics, 1994.
10. Artale A., Franconi E. A Survey of Temporal Extensions of Description Logics. In:Annals of Mathematics and Artificial Intelligence, Vol.30, No.1-4, pp.171-210.
11. Baader F., Calvanese D., McGuinness D., Nardi D., Patel-Schneider P. The Description Logic Handbook. Theory, Implementation and Applications. Cambridge Univ. Press, 2003. – 555 pp.
12. Pnueli A. The temporal logic of programs. In: Proc. 18th Annual Symposium on Foundations of Computer Science (Providence, R.I.), IEEE, New York, 1977, pp.46-57.
13. Gabbay D., Pnueli A., Shelah S., Stavi J. On the temporal analysis of fairness. In: Proc. 7th Annual ACM Symposium on Principles of Programming Languages. ACM, New York, pp.163-173.
14. Mendelson E. Introduction to Mathematical Logic. M.: Nauka, 1971. – 320 pp. (in Russian)
15. Keberle N.G. Properties of Propositional Metric Temporal Calculus For Description Of Evolving Conceptualization. In: Bulletin of the Dniepropetrovsk National University, – 2006, to appear.
16. Feys R. Modal Logic. M.: Nauka, – 1974. – 520 pp. (in Russian).